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Three-Loop Four-Point Correlator in $N = 4$ SYM

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Abstract

We explicitly compute the complete three-loop ($\mathrm{O}(g^4)$) contribution to the four-point function of chiral primary current-like operators $\langle \tilde{q}^2 q^2 \tilde{q}^2 q^2 \rangle$ in any finite $N = 2$ SYM theory. The computation uses $N = 2$ harmonic supergraphs in coordinate space. Dramatic simplifications are achieved by a double insertion of the $N = 2$ SYM linearized action, and application of superconformal covariance arguments to the resulting nilpotent six-point amplitude. The result involves polylogarithms up to fourth order of the conformal cross ratios. It becomes particularly simple in the $N = 4$ special case.

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Correlators of gauge invariant composite operators are natural objects to study in finite super-Yang-Mills theories, since they are strongly constrained by superconformal invariance. These constraints have recently been actively investigated using both abstract methods [1]-[5] and the operator product expansion [2, 6, 7]. Additional motivation is provided by the AdS/CFT correspondence conjecture [8, 9, 10], which relates correlators of chiral primary operators in $N = 4$ super-Yang-Mills theory to correlators in AdS supergravity [11]-[19]. This relation involves the SYM correlators at strong coupling, so that its verification presently relies mostly (although not exclusively [20]) on non-renormalization theorems [16],[21]-[28].

On the CFT side, the correlator of four $N = 4$ stress tensors is the simplest one which can be built from chiral primary operators, and is *not* subject to any known non-renormalization theorem. The quantum corrections to this correlator have so far been investigated to lowest-order in the perturbative (two-loop) [29]-[32] and non-perturbative (one-instanton) [20] sectors. From the point of view of the operator product expansion approach it is also of interest to know the singularity structure of this correlator at coincidence points [7, 33].

In this paper we a make a step further by computing it at the next, three-loop level. Our computation is not restricted to $N = 4$ SYM theory but concerns the correlator of four bilinear (current-like) hypermultiplet matter composite operators in $N = 2$ SYM. There are three reasons for staying at the $N = 2$ level: i) there is no known off-shell formulation of $N = 4$ SYM, so the best way to carry out a quantum calculation is to reformulate the theory in $N = 2$ harmonic superspace [34] and then use the efficient supergraph technique available there [35]; ii) as shown in [29], knowing the result in the $N = 2$ matter sector and using the $SU(4)$ R symmetry of the $N = 4$ theory one can easily reconstruct the complete amplitude for four $N = 4$ stress tensors; iii) it is not impossible that many of the expected exceptional features of four-dimensional CFT are shared by all finite $N = 2$ theories [36, 37].

The correlator we consider is made out of hypermultiplets $q^+, \tilde{q}^+$ and has the form

$$G \quad = \quad \langle \mathrm{tr}(\tilde{q}^2)\mathrm{tr}(q^2)\mathrm{tr}(\tilde{q}^2)\mathrm{tr}(q^2)\rangle \equiv \langle \mathcal{O}_1 \cdots \mathcal{O}_4 \rangle \tag{1}$$

In the $SU(2)$-covariant harmonic superspace formalism [34] this hypermultiplet is described off shell by a Grassmann analytic superfield $q^+(x_A, \theta^+, \bar{\theta}^+, u^\pm)$. The harmonic variables are defined as $SU(2)$ matrices,

$$u \in SU(2) \quad \Rightarrow \quad u_i^- = (u^{+i})^* \;, \quad u^{+i}u_i^- = 1 \tag{2}$$

The Grassmann variables

$$\theta^{+\alpha} = u_i^+ \theta^{i\alpha}, \quad \bar{\theta}^{+\dot{\alpha}} = u_i^+ \bar{\theta}^{i\dot{\alpha}} \tag{3}$$

are $SU(2)$-invariant $U(1)$ projections of the full superspace ones $\theta^i, \bar{\theta}^i$. The coordinates $x_A^{\alpha\dot{\alpha}} = x^{\alpha\dot{\alpha}} - 4i\theta^{(i\,\alpha}\bar{\theta}^{j)\,\dot{\alpha}}u_i^+u_j^-$ together with $\theta^+, \bar{\theta}^+$ and $u^\pm$ span the G-analytic superspace closed under the full $N = 2$ superconformal group (see [34, 38] for details). In order for the theory to be finite, the matter hypermultiplets q^+ must be in a representation r of the gauge group such that $C(r) = C_2(G)$ [36, 37] [2].

[2] We denote the generators in this representation by t^a, and $\mathrm{tr}(t^a t^b) = C(r)\delta^{ab}$, $t^a t^a = C_2(r) \cdot \mathbb{1}$.

A direct calculation of this correlator at three loops using standard component or $N = 1$ superfield techniques would be, at the present state-of-the-art, a prohibitively difficult task. In the following we will perform it in a rather roundabout way, using the new set of coordinate-space Feynman rules for $N = 2$ harmonic superspace given in [31], together with a fortuitous combination of Intriligator's insertion trick and knowledge about the construction of superconformal invariants which was built up in [23, 3, 29, 30, 31, 26, 5].

Intriligator's trick, introduced in the present context in [18], allows us to write the three-loop ($\mathrm{O}(g^4)$) contribution to this correlator in terms of a double insertion of the linearized SYM action,

$$G^{3-\text{loop}} \quad \sim \quad \int d^4x_5 d^4\theta_5 \int d^4x_6 d^4\theta_6 \Big\langle \mathcal{O}_1 \cdots \mathcal{O}_4 \mathrm{tr} W_5^2 \mathrm{tr} W_6^2 \Big\rangle \tag{4}$$

where the integrals are over $N = 2$ chiral superspace. This representation can be derived either by a simple path integral manipulation [18, 31], or diagrammatically by showing that this expression differs from the original set of Feynman diagrams only by a simultaneous change of gauge for all SYM propagators [39]. One advantage of this representation is that it leads, due to the fact that W is chiral and q^+ is G-analytic, to severe restrictions on the possibilities for building superconformal invariants.

We have already successfully employed this trick in the analogous two-loop calculation. There one makes a single insertion and deals with the five-point correlator $\langle \mathcal{O}_1 \cdots \mathcal{O}_4 \mathrm{tr} W_5^2 \rangle$. It is easy to see that superconformal invariance alone completely determines its Grassmann dependence [26, 5]. Indeed, the correlator must be a superconformal covariant having the R weight of four left-handed θ's (in order to mach that of the chiral measure $d^4x_5 d^4\theta_5$). This means it has to be made out of combinations of $\theta^+_{1,\dots,4}$ and θ^i_5 invariant under the shift part of the superconformal transformations. Since the latter involve 4 left-handed odd parameters, there exist only two such combinations ξ_{123} and ξ_{124} given below [2, 3, 26]:

$$\begin{aligned} \xi^{\dot\alpha}_{12m} &\equiv (12)\rho^{\dot\alpha}_m + (2m)\rho^{\dot\alpha}_1 + (m1)\rho^{\dot\alpha}_2, \quad m = 3,4, \\ \rho^{\dot\alpha}_a &\equiv \theta_{5a\alpha} \frac{x^{\alpha\dot\alpha}_{5a}}{x^2_{5a}}, \quad a = 1,\dots,4, \\ \theta^{\alpha}_{5a} &\equiv u^+_{ai}\theta^{i\alpha}_5 - \theta^{+\alpha}_a \end{aligned} \tag{5}$$

($(12) = u^{+i}_1 u^+_{2i}$, $x_{5a} = x_{L5} - x_{Aa}$). Then it is clear that the correlator must have the form

$$\Big\langle \mathcal{O}_1 \cdots \mathcal{O}_4 \mathrm{tr} W_5^2 \Big\rangle \quad = \quad \xi^4 F(x,u) + \mathrm{O}(\theta^5\bar\theta) \tag{6}$$

where $\xi^4 \equiv (12)^{-2}\xi^2_{123}\xi^2_{124}$, and the factor $F(x,u)$ is a conformal covariant depending only on the space-time and harmonic variables. The latter *is not predicted by invariance alone* and had to be determined by an explicit graph calculation. It was rather surprising to find out that the result was a *rational function* of the space-time variables. Then, after the single integration over

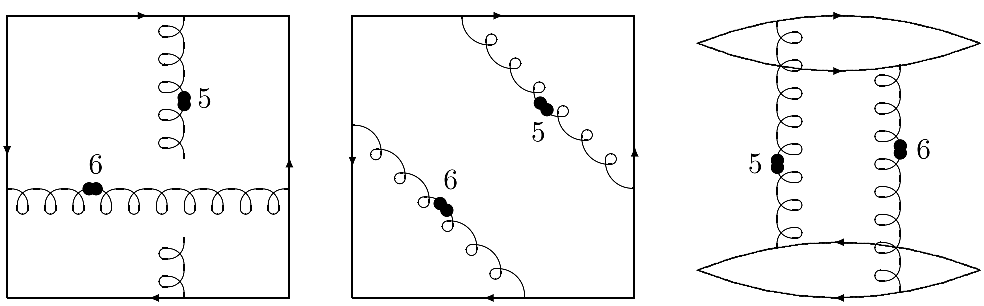
the insertion point $\int d^4x_5$ we found that the entire amplitude was expressed in terms of the one-loop scalar box integral.

Extending the same argument based on counting the R weight and the number of independent shift-invariant combinations of left-handed θ's to the three-loop case, one finds that superconformal invariance constrains the integrand to be of the form

$$\left\langle \mathcal{O}_1 \cdots \mathcal{O}_4 \mathrm{tr} W_5^2 \mathrm{tr} W_6^2 \right\rangle \quad = \quad \xi^4 \psi^4 G(x,u) + \mathrm{O}(\theta^9 \bar{\theta}) \qquad (7)$$

where ξ^4 is the same as above, and ψ^4 the corresponding covariant referring to point '6', built from $\sigma_a^{\dot{\alpha}} \equiv \theta_{6a\alpha} \frac{x_{6a}^{\alpha\dot{\alpha}}}{x_{6a}^2}$. Once again, the purpose of the graph calculation is to determine the factor $G(x,u)$.

The knowledge of the dependence on the odd coordinates in (7) is extremely useful, since it allows one to concentrate on one typical term in the expansion of the nilpotent covariant while doing the explicit graph calculation. Thus, in the two-loop calculation [31] a very substantial simplification was reached by setting the analytic Grassmann variables to zero, $\theta_1^+ = \ldots = \theta_4^+ = 0$, and keeping only the chiral θ_5. At the three-loop level, it turns out that even more dramatic simplifications can be achieved by keeping only the external Grassmann variables, and setting to zero both θ_5 and θ_6. Further, we are only interested in the leading term in (7), so we can set all right-handed $\bar{\theta}^+_{1,\ldots,4} = 0$. Then the whole correlator becomes proportional to $\theta_1^{+2}\theta_2^{+2}\theta_3^{+2}\theta_4^{+2}$. Now, this nilpotent factor absorbs the complete U(1) charge, so its coefficient is chargeless and hence harmonic independent [3]. This allows one to identify all harmonic variables, $u_1 = \ldots = u_4$. The combination of all this turns out to have the effect of eliminating all Feynman diagrams except those with exactly one interaction vertex along every matter line. Up to permutations, this leaves only the three diagrams depicted in fig. 1. (In particular, all diagrams involving gauge self-interactions drop out.)



A B C

Figure 1

These diagrams involve only the "building block" shown in fig. 2.

[3]This argument is based on harmonic analyticity. It should be stressed that the latter is not an assumption but can easily be proven by examining the complete set of 3-loop graphs [39].

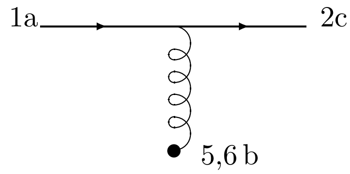


Figure 2

This building block, which we denote by $I_{5,6}$, is already known from the two-loop calculation [31]. With the stated specializations, $\theta_{5,6} = \bar{\theta}_i^+ = u_i - u_j = 0$, the expression obtained there can be rewritten in terms of the variables $\rho_i(\sigma_i)$ as follows,

$$I_5 \;=\; \frac{ig(t^b)^{ac}}{(2\pi)^4 x_{12}^2}\,(\rho_1-\rho_2)^2 \tag{8}$$

Note that here the integration over the interaction point has already been performed. We can thus immediately write down the contributions of all graphs to the six-point correlator. The result reads, after some fierzing (up to an overall factor),

$$\begin{aligned}
A &= \frac{C_A}{x_{12}^2x_{23}^2x_{34}^2x_{41}^2}\Big[\tau_{13}\tau_{24}-\tau_{12}\tau_{34}-\tau_{14}\tau_{23}-\tau_{24}(\rho_1^2\sigma_3^2+\rho_3^2\sigma_1^2)-\tau_{13}(\rho_2^2\sigma_4^2+\rho_4^2\sigma_2^2)\\
&\quad +\tau_{12}(\rho_3^2\sigma_4^2+\rho_4^2\sigma_3^2)+\tau_{34}(\rho_1^2\sigma_2^2+\rho_2^2\sigma_1^2)+\tau_{14}(\rho_2^2\sigma_3^2+\rho_3^2\sigma_2^2)+\tau_{23}(\rho_1^2\sigma_4^2+\rho_4^2\sigma_1^2)\Big]\\
B &= \frac{C_B}{x_{12}^2x_{23}^2x_{34}^2x_{41}^2}\Big[\tau_{13}(\rho_2^2\sigma_4^2+\rho_4^2\sigma_2^2)+\tau_{24}(\rho_1^2\sigma_3^2+\rho_3^2\sigma_1^2)\Big]\\
C &= C_C\Big[\frac{\tau_{12}\tau_{34}}{x_{12}^4x_{34}^4}+\frac{\tau_{14}\tau_{23}}{x_{14}^4x_{23}^4}\Big]
\end{aligned} \tag{9}$$

where $\tau_{ij} \equiv 4(\rho_i\rho_j)(\sigma_i\sigma_j)+\rho_i^2\sigma_j^2+\rho_j^2\sigma_i^2$. The color factors $C_{A,B,C}$ are

$$\begin{aligned}
C_A &= d(G)C(r)\Big[C_2(r)-\frac{1}{2}C_2(G)\Big]\\
C_B &= d(G)C(r)C_2(r)\\
C_C &= d(G)C^2(r)
\end{aligned} \tag{10}$$

($d(G)$ denotes the dimension of the gauge group). In terms of the variables τ_{ij} the six-point nilpotent covariant reads

$$\begin{aligned}
\xi^4\psi^4 &= (12)^2(34)^2\tau_{14}\tau_{23}+(14)^2(23)^2\tau_{12}\tau_{34}\\
&\quad +(12)(23)(34)(41)\Big[\tau_{13}\tau_{24}-\tau_{12}\tau_{34}-\tau_{14}\tau_{23}\Big]
\end{aligned} \tag{11}$$

For $\theta_5 = \theta_6 = 0$ all these expressions are easy to evaluate, since then

$$\rho_i^2 = \frac{\theta_i^{+2}}{x_{i5}^2}, \quad \sigma_i^2 = \frac{\theta_i^{+2}}{x_{i6}^2}, \quad \tau_{ij} = \theta_i^{+2}\theta_j^{+2}\frac{x_{ij}^2 x_{56}^2}{x_{i5}^2 x_{i6}^2 x_{j5}^2 x_{j6}^2} \tag{12}$$

Thus the six-point covariant becomes

$$\xi^4\psi^4 \mid_{\theta_{5,6}=0} = \theta_1^{+2}\theta_2^{+2}\theta_3^{+2}\theta_4^{+2}\frac{x_{56}^4 R'}{\prod_{i=1}^4 x_{i5}^2 x_{i6}^2} \tag{13}$$

where

$$\begin{aligned} R' &= (12)^2(34)^2 x_{14}^2 x_{23}^2 + (14)^2(23)^2 x_{12}^2 x_{34}^2 \\ &+ (12)(23)(34)(41)\Big[x_{13}^2 x_{24}^2 - x_{12}^2 x_{34}^2 - x_{14}^2 x_{23}^2\Big] \end{aligned} \tag{14}$$

The coefficient function $G(x,u)$ can now be determined by comparing the $\theta_1^{+2}\theta_2^{+2}\theta_3^{+2}\theta_4^{+2}$ coefficients of the six-point correlator and the covariant. Once this is done we know this correlator in covariant form, so that we can now return to the "opposite" frame where $\theta_1^+ = \ldots = \theta_4^+ = 0$. Here the covariant also looks very simple,

$$\xi^4\psi^4 \mid_{\theta^+_{1,2,3,4}=0} = \theta_5^4\theta_6^4\frac{R'^2}{\prod_{i=1}^4 x_{i5}^2 x_{i6}^2} \tag{15}$$

It is remarkable and rather unexpected that this result only involves a *rational* function of the space-time variables, just like in the two-loop calculation. The Grassmann integrations being trivial ($\theta_{5,6}^4$ are just chiral delta functions), one is left with the space-time integrals $\int dx_5 \int dx_6$. Only two different integrals appear, namely the standard one-loop box integral

$$h^{(1)}(x_1,x_2,x_3,x_4) \equiv \int \frac{dx_5}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2} = -\frac{i\pi^2}{x_{13}^2 x_{24}^2}\Phi^{(1)}\Big(\frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}\Big) \tag{16}$$

and one other conformally invariant integral, e.g.,

$$\begin{aligned} h_{12}^{(2)}(x_1,x_2,x_3,x_4) &\equiv x_{12}^2 \int dx_5 \int dx_6 \, \frac{1}{x_{15}^2 x_{25}^2 x_{35}^2 x_{56}^2 x_{16}^2 x_{26}^2 x_{46}^2} \\ &= \frac{(i\pi^2)^2}{x_{12}^2 x_{34}^2}\Phi^{(2)}\Big(\frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2}, \frac{x_{14}^2 x_{23}^2}{x_{12}^2 x_{34}^2}\Big) \end{aligned} \tag{17}$$

The first one is well-known [40], while the second one can be rewritten in terms of the two-loop (momentum space) double box integral, calculated in [41]. The functions $\Phi^{(1,2)}$ are the first two elements of the infinite series of conformal "multi-ladder" functions introduced by Davydychev and Ussyukina [42, 43]. They can be written in terms of polylogarithms Li_n as follows [41],

$$
\begin{aligned}
\Phi^{(1)}(x,y) &= \frac{1}{\lambda}\left\{2\Big(\mathrm{Li}_2(-\rho x)+\mathrm{Li}_2(-\rho y)\Big)+\ln\frac{y}{x}\ln\frac{1+\rho y}{1+\rho x}+\ln(\rho x)\ln(\rho y)+\frac{\pi^2}{3}\right\} \\
\Phi^{(2)}(x,y) &= \frac{1}{\lambda}\left\{6\Big(\mathrm{Li}_4(-\rho x)+\mathrm{Li}_4(-\rho y)\Big)+3\ln\frac{y}{x}\Big(\mathrm{Li}_3(-\rho x)-\mathrm{Li}_3(-\rho y)\Big)\right. \\
&\quad +\frac{1}{2}\ln^2\frac{y}{x}\Big(\mathrm{Li}_2(-\rho x)+\mathrm{Li}_2(-\rho y)\Big)+\frac{1}{4}\ln^2(\rho x)\ln^2(\rho y) \\
&\quad \left.+\frac{1}{2}\pi^2\ln(\rho x)\ln(\rho y)+\frac{1}{12}\pi^2\ln^2\frac{y}{x}+\frac{7}{60}\pi^4\right\}
\end{aligned}
\tag{18}
$$

where $\lambda(x,y)=\sqrt{(1-x-y)^2-4xy}$, $\rho(x,y)=2(1-x-y+\lambda)^{-1}$. The final result after integration over points $5,6$ is reached by replacing, in eq. (9),

$$
\begin{aligned}
\tau_{12}\tau_{34} &\rightarrow R' x_{12}^2 x_{34}^2\Big(h^{(1)}(x_1,x_2,x_3,x_4)\Big)^2, \\
\tau_{12}(\rho_3^2\sigma_4^2+\rho_4^2\sigma_3^2) &\rightarrow 2R' h_{12}^{(2)}(x_1,x_2,x_3,x_4)
\end{aligned}
\tag{19}
$$

etc. This result holds for any finite $N=2$ SYM theory. It considerably simplifies if one specializes to the $N=4$ case, where $(t^b)^{ac}=if^{abc}$, and to the gauge group $SU(N_c)$. Here all three contributions can be added up, leading to

$$
\begin{aligned}
\int_5\int_6\Big\langle\mathcal{O}_1\cdots\mathcal{O}_4\mathrm{tr}W_5^2\mathrm{tr}W_6^2\Big\rangle\,|_{\theta^+_{1,2,3,4}=0} \;\sim\; & \frac{(N_c^2-1)N_c^2R'}{x_{12}^2x_{23}^2x_{34}^2x_{41}^2}\Big[(x_{12}^2x_{34}^2+x_{13}^2x_{24}^2+x_{14}^2x_{23}^2)(h^{(1)})^2 \\
& +4(h_{12}^{(2)}+h_{13}^{(2)}+h_{14}^{(2)})\Big]
\end{aligned}
\tag{20}
$$

The details of this calculation will be given elsewhere [39], as well as a discussion of the result. Here we only mention one important point. We have checked that our result exhibits a singularity of the type $\log^2 x_{12}^2$ in the coincidence limit $x_{12}\to 0$, exactly as predicted in [7, 33].

It should also be mentioned that our three-loop result already dispenses with a speculation made in [30]. There it was suggested that the unexpected absence of three- and quadrilogarithms in the two-loop result may be related to the fact that the complete tree-level result for the corresponding axion/dilation amplitudes in AdS supergravity can be represented in terms of logarithms and dilogarithms of the conformal cross ratios [17]. With hindsight, the simplicity of the two-loop result is just a consequence of the fact that, apart from the standard box integral $h^{(1)}$, no other finite and conformally invariant scalar integral exists at this level.

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References

[1] P.S. Howe and P.C. West, Int. J. Mod. Phys. **A14** (1999) 2659, hep-th/9509140; Phys. Lett. **B389** (1996) 273, hep-th/9607060; *Is $N = 4$ Yang-Mills soluble?* (1996) in Moscow 1996, hep-th/9611074; Phys. Lett. **B400** (1997) 307, hep-th/9611075.

[2] H. Osborn, Ann. Phys. **272** (1999) 243, hep-th/9808041.

[3] B. Eden, P.S. Howe and P.C. West, Phys. Lett. **B463** (1999) 19, hep-th/9905085.

[4] S.M. Kuzenko and S. Theisen, Class. Quant. Grav. **17** (2000) 665, hep-th/9907107.

[5] B. Eden, P.S. Howe, A. Pickering, E. Sokatchev, and P.C. West, KCL-MTH-99-46, hep-th/0001138.

[6] K. Intriligator and W. Skiba, Nucl. Phys. **B559** (1999) 165, hep-th/9905020.

[7] M. Bianchi, S. Kovacs, G. Rossi, and Y.S. Stanev, ROM2F/99/9, hep-th/9906188.

[8] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231, hep-th/9711200.

[9] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. **B428** (1998) 105, hep-th/9802109.

[10] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253, hep-th/9802150.

[11] W. Mück and K.S. Viswanathan, Phys. Rev. **D58** (1998) 041901, hep-th/9804035.

[12] D.Z. Freedman, S. Mathur, A. Matusis, and L. Rastelli, Nucl. Phys. **B546** (1999) 96, hep-th/9804058.

[13] S. Lee, S. Minwalla, M. Rangamani, and N. Seiberg, Adv. Theor. Math. Phys. **2** (1998) 697, hep-th/9806074.

[14] G. Chalmers, H. Nastase, K. Schalm, and R. Siebelink, Nucl. Phys. **B540** (1999) 247, hep-th/9805105.

[15] H. Liu and A.A. Tseytlin, Phys. Rev. **D59** (1999) 086002, hep-th/9807097.

[16] E. D'Hoker, D.Z. Freedman and W. Skiba, Phys. Rev. **D59** (1999) 045008, hep-th/9807098.

[17] E. D'Hoker, D.Z. Freedman, S. Mathur, A. Matusis and L. Rastelli, Nucl. Phys. **B562** (1999) 353, hep-th/9903196.

[18] K. Intriligator, Nucl. Phys. **B551** (1999) 575, hep-th/9811047.

[19] S. Penati, A. Santambrogio, and D. Zanon, Bicocca-FT-99-30, hep-th/9910197.

[20] M. Bianchi, M. Green, S. Kovacs, and G. Rossi, JHEP **08** (1998) 13, hep-th/9807033.

[21] S.S. Gubser and I. Klebanov, Phys. Lett. **B413** (1997) 41, hep-th/9708005.

[22] D. Anselmi, D.Z. Freedman, M. Grisaru, and A. Johansen, Nucl. Phys. **B526** (1998) 543, hep-th/9708042.

[23] P.S. Howe, E. Sokatchev and P.C. West, Phys. Lett. **B444** (1998) 341, hep-th/9808162.

[24] A. Petkou and K. Skenderis, hep-th/9906030.

[25] E. D'Hoker, D.Z. Freedman, S.D. Mathur, A. Matusis, and L. Rastelli, MIT-CTP-2893, hep-th/9908160.

[26] B. Eden, P.S. Howe, C. Schubert, E. Sokatchev, and P.C. West, Phys. Lett. **B472** (2000) 323, hep-th/9910150.

[27] M. Bianchi and S. Kovacs, ROM2F/99/31, hep-th/9910016.

[28] J. Erdmenger and M. Perez-Victoria, MIT-CTP-2933, hep-th/9912250.

[29] B. Eden, P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, Nucl. Phys. **B557** (1999) 355, hep-th/9811172.

[30] B. Eden, P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, Phys. Lett. **B466** (1999) 20, hep-th/9906051.

[31] P.S. Howe, C. Schubert, E. Sokatchev, and P.C. West, KCL-MTH-99-41, Nucl. Phys. **B** (in print), hep-th/9910011.

[32] F. Gonzalez-Rey, I. Park and K. Schalm, Phys. Lett. **B448** (1999) 37, hep-th/9811155.

[33] E. D'Hoker, S. Mathur, A. Matusis and L. Rastelli, UCLA-99-TEP-46, hep-th/9911222.

[34] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky, and E. Sokatchev, Class. Quant. Grav. **1** (1984) 469.

[35] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, Class. Quant. Grav. **2** (1985) 601; Class. Quant. Grav. **2** (1985) 617.

[36] P.S. Howe, K. Stelle, P. West, Phys. Lett. **124 B** (1983) 55.

[37] J.P. Derendinger, S. Ferrara, A. Masiero, Phys. Lett. **143 B** (1984) 133.

[38] A. Galperin, E. Ivanov, V. Ogievetsky, and E. Sokatchev, in *Quantum Field Theory and Quantum Statistics*, ed. I. Batalin, C.J. Isham, G. Vilkovisky, vol. 2, p. 233, Adam Hilger, Bristol, 1987.

[39] B. Eden, C. Schubert, and E. Sokatchev, in preparation.

[40] G. 't Hooft and M. Veltman, Nucl. Phys. **B153** (1979) 365.

[41] N.I. Ussyukina and A.I. Davydychev, Phys. Lett. **B 298** (1993) 363.

[42] N.I. Ussyukina and A.I. Davydychev, Phys. Lett. **B 305** (1993) 136.

[43] D.J. Broadhurst, Phys. Lett. **B 307** (1993) 132.